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INDEPENDENT ANALYSIS OF μ_δ DATA OF ILS AND INDLS CATALOGS TO OBTAIN THE SPIN OF THE BRIGHT GAIA DR2 REFERENCE FRAME

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SUMMARY: The Gaia DR2 reference frame should be without relative rotation to the quasars (QSOs) and consistent with the International Celestial Reference System (ICRS). For the faint part of DR2 (stars with Gaia magnitude $G \geq 16$) that task was done via Gaia's observations of QSOs ($G \geq 17$ mag), but the bright DR2 ($G \leq 13$ mag) is difficult to validate and it rotates relative to the faint DR2 at rate of the order of 0.1 mas/yr. Very bright DR2 stars ($G \leq 6$ mag) mostly have inferior astrometry. Here, the aim is to determine two spin components (ω_X and ω_Y) of the bright DR2 using International Latitude Service (ILS, for 387 stars) and independent latitude stations (INDLS, for 682 stars) catalogs of proper motion in declination μ_δ; both are referred to the Hipparcos reference frame and their stars are mostly from 4 to 8 mag in the V-band (critical part of DR2). Also, using the new Hipparcos (NHIP) values μ_δ for ILS and INDLS stars, we can see that the merit of the ILS and INDLS is the long time baseline ($\Delta t \approx 90$ years) important for μ_δ because the standard deviation of μ_δ is opposite to Δt. Applying the least squares method (LSM) to the differences of μ_δ between two catalogs (ILS−DR2, INDLS−DR2, etc.), our results support the mentioned spin. The 3σ criterion and Tukey's fences method were used to reject some stars, the Abbe criterion to explain the variability in ILS−DR2 and other μ_δ differences, and the Shapiro-Wilk test to check the standard distribution of differences. The obtained ω_Y is significant at the 2σ level, and the ILS and INDLS catalogs could be useful for validation of the bright reference frame of Gaia DR2.

Key words. Catalogs – Surveys – Proper motions – Reference systems

1. INTRODUCTION

The quasars (QSOs) define a kinematic reference frame (non-rotating one) because their cosmological distances are the reason for their nearly zero proper motions. Some of QSOs are visible in both optical and radio domains. Using very long baseline interferometry data (VLBI) their accurate positions are obtained in radio domain and included into the International Celestial Reference Frame – ICRF (Ma et al. 1998). The optical counterparts of some QSOs (which were observed with Gaia) are of importance for astrometry because they are useful to align the Gaia Celestial Reference Frame (Gaia CRF) with the ICRF (Gaia Collaboration et al. 2018b). About that link, the QSOs with optical counterparts (which define Gaia CRF) are: faint objects (mostly with Gaia magnitude $G \geq 17$ which is close to the faint Gaia DR2 stars), in accordance with the faint part of Gaia DR2 (stars with $G \geq 16$ mag), in disadvantage for

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the bright Gaia DR2 ($G \leq 13$ mag), with sky distribution different from that for stars, etc.

There is an indication that the bright reference frame of the second Gaia data release – Gaia DR2 rotates with respect to the faint part of DR2 (the quasars based one) by the order of 0.1 mas/yr (Lindegren 2020b); the spin components are ω_X, ω_Y, and ω_Z. Also, there is the Lindegren et al. (2018) result that the DR2 stars with $G \leq 6$ mag mostly have inferior astrometry. A set of proper motions which are calculated from the position differences between DR2 and the Hipparcos (the epoch difference is about 24 years) also shows some systematic errors of the bright DR2 proper motions (Brandt 2018); this is equivalent to an inertial rotation of the bright DR2 reference frame.

To check the quality of the bright part of the Gaia reference frame, an independent and accurate set of data is of importance. In the paper Lindegren (2020b) this could be the VLBI technique and observations of radio stars. Here, the goal is to present the possibility of the International Latitude Service - ILS and independent latitude stations - INDLS catalogs of μ_δ for verification of Gaia's bright reference frame. Both catalogs (the ILS and INDLS) are referred to the Hipparcos reference frame but their merit is the long time baseline important for μ_δ because $\Delta t \approx 90$ yr and the standard deviation of μ_δ is inversely proportional to Δt. In the case of the Hipparcos data, the μ_δ values are affected by the systematic errors of double and multiple stars because $\Delta t < 4$ yr. Both catalogs mostly contain stars from 4 to 8 mag in the V-band (critical part of DR2) and since it is difficult to valdite the spin of the bright DR2, that is why we compare our μ_δ with suitable values of common stars in the bright part ($G \leq 13$ mag) of Gaia DR2 catalog.

The Section 2 presents the main information about the Hipparcos, Gaia DR2, ILS, and INDLS data. The steps of the ω_X and ω_Y calculations are given in Section 3. The results and discussion are given in Section 4. Conclusions are in Section 5.

2. HIPPARCOS, GAIA DR2, ILS, AND INDLS DATA

Behind both satellite missions (Hipparcos and Gaia) is the European Space Agency (ESA). In the paper Damljanović and Taris (2019) we used the Hipparcos catalog data (ESA 1997), new Hipparcos catalog data (van Leeuwen 2007), Gaia DR2, and ILS to compare μ_δ values (of common stars) between each other. Also, in Damljanović and Taris (2019) the main steps of the ILS construction are presented. Similarly, this was done for the INDLS catalog in the paper Damljanović (2020). The main task is to obtain the spin values ω_X and ω_Y (see Table 1) of the bright DR2 reference frame using the ILS and INDLS μ_δ data. The component ω_Z of the spin is not possible to calculate using μ_δ, but from the other investigations $\omega_Z \approx 0$ mas/yr (Lindegren 2020a).

The Hipparcos reference frame has been linked to the ICRF, and the Hipparcos catalog is the optical counterpart of the ICRF (Kovalevsky et al. 1997). The Hipparcos observation period was less than four years ($\Delta t < 4$ yr); it could be a problem for accurate positions and proper motions of double and multiple stars. There are 118218 stars (with $V \leq 12$ mag, but mostly $7 \leq V \leq 9$ mag), and their errors are: about 1 mas in position and parallax (at J1991.25), about 1 mas/yr in the proper motion; the errors of proper motion are larger in the case of double and multiple than for single stars (Vondrák et al. 1998). After a new reduction of raw Hipparcos observations, the new Hipparcos catalog appeared with improved: coordinates, proper motions, and parallaxes of stars. The new astrometric data are better by a factor of 2.2 in total weight, and by up to a factor of 4 for stars with $V \leq 8$ mag (van Leeuwen 2007). Also, the ILS and INDLS stars are brighter than 8 V-mag (Damljanović and Taris 2019, Damljanović 2020). To get more accurate values of positions and proper motions, it is useful to combine the ground-based and satellite data; the results are: ARIHIP, the Earth Orientation Catalog – EOC (Vondrák and Ron 2003), ILS (Damljanović and Taris 2019), INDLS (Damljanović 2020), etc.

The Gaia satellite was launched in December 2013 (Gaia Collaboration et al. 2016b), and in July 2014 it started its operation phase to collect astronomical data for more than one billion stars and for about 500000 QSOs (with $G \leq 20.7$ mag): positions, proper motions, and parallaxes (Prusti 2012). Until now, there are only a few solutions (Gaia Collaboration et al. 2016a, Lindegren et al. 2018): Gaia DR1 (the first one is based on the first 14 months of Gaia observations and appeared in September 2016), DR2 (the second one, its results are from the first 22 months of observations, and it appeared in April 2018), and EDR3 (the early third release with results using the first 34 months of observations, and it appeared in December 2020). There are about 1.69 billion sources in DR2 (with $3 \leq G \leq 21$ mag): about 1.33 billion sources with five astrometric parameters (coordinates, proper motions, and parallaxes), and about 0.36 billion faint objects (with the approximate positions); the reference epoch is J2015.5. The Gaia DR2 optical reference frame (the faint part of it with $G \geq 16$ mag) is aligned with the ICRS via quasars (Lindegren et al. 2018). The median uncertainty in coordinates and parallax is about 0.04 mas for sources with $G < 14$ mag at J2015.5 (it is 0.05 mas/yr in the proper motion). In the Gaia DR2 (Gaia Collaboration et al. 2018a) there are about 560000 QSOs, and 2820 QSOs are from a prototype version of ICRF3 (Jacobs et al. 2018); these objects and their positions define the optical reference frame (Gaia CFR2) at epoch J2015.5. Bright stars (with $G \leq 6$ mag) mostly have inferior astrometry due to the calibration issues (Lindegren et al. 2018) and a few times bigger error in the proper motion. In the paper Damljanović and Taris (2019), using the ILS catalog it was shown that for these DR2 stars there are some systematic errors in line with the magnitude and color index of a star; similar results were presented in the paper Damljanović (2020) about the INDLS stars.

The Gaia DR2 is an important step in realization of the future Gaia reference frame, and useful remarks about DR2 could improve the next release. Ideally, the Gaia reference frame should coincide with the International Celestial Reference System (ICRS) for both bright and faint sources, but for the DR2 bright sources there are no Gaia's direct observations of QSOs and there is no comparison of optical Gaia DR2 data with accurate VLBI data (of the same objects visible in optical and radio domains). The quality of the reference frame is better for the DR2 faint part than for the DR2 bright part. Because of this, we present an independent contribution to a possible validation of Gaia astrometry (DR2 data) using the ILS (with 387 stars) and INDLS (with 682 stars) catalogs.

The ILS catalog is based on zenith telescope data (Yumi and Yokoyama 1980) spanning a time baseline of about 80 years ($\Delta t \approx 80$ yr) during the period of 1899.7-1979.0 (more than 90 years including the Hipparcos point at 1991.25); these long history data are of importance to get accurate proper motions. The error of μ_δ is proportional to $1/\Delta t$ (Eichhorn 1974). Moreover, there were seven ILS stations (Carloforte, Cincinnati, Gaithersburg, Kitab, Mizusawa, Tschardjui, and Ukiah), and many observations of the same Talcott star pairs (from a few to a few hundred times per star and per year). First, the reduction of these stars to the Hipparcos reference system was done (Vondrák et al. 1998); second, the original catalog of μ_δ (the ILS catalog) for 387 bright stars was constructed (Damljanović and Pejović 2006, Damljanović and Taris 2019); third, some comparison results (Damljanović and Taris 2019) of the four catalogs by pairs (the ILS, Hipparos – HIP, new Hipparcos – NHIP, and Gaia DR2) were presented, and fourth, the differences in μ_δ between pairs of catalogs to characterize the μ_δ errors was analyzed for these catalogs with a special focus on the Gaia DR2 and ILS catalogs (Damljanović and Taris 2019). A similar procedure was done for construction of the INDLS catalog based on seven independent latitude stations: Belgrade, Blagoveschtschensk, Irkutsk, Mizusawa, Poltava, Pulkovo, and Warsaw; more about it in Damljanović (2020). In the present paper, the subject is the spin of the bright part of Gaia DR2 using the ILS and INDLS catalogs.

Due to the specific Horrebow-Talcott method (Yumi and Yokoyama 1980) of the measured star pairs of ILS and INDLS stars, it is difficult to determine μ_δ for each single star (Vondrák 2004), but the original method was developed (Damljanović 2007) and also the Hipparcos data were introduced into the calculation; the least squares method (LSM) was used. Distribution of the ILS stars via coordinates and V magnitude is: $0^h < \alpha \le 24^h$, $20^\circ \le \delta \le 60^\circ$, and $4 \le V \le 8$ (mostly, from 6 mag to 7 mag). It is similar for the INDLS stars, but $30^\circ \le \delta \le 80^\circ$.

The mean accuracy of ILS is 0.21 mas/yr; it is 0.144 for the same DR2 stars, 0.58 for HIP, and 0.36 for NHIP. The verification of the ILS data is carried out by applying the F-test. Using the 3σ criterion 9 stars were rejected (see Table 2), and for $n = 378$ ILS stars the test statistic is $F = S_1^2/S_2^2$. If $F \ge F_{n-1,n-1;0.05}$ one concludes that: S_2^2 is smaller than S_1^2, where $F_{377,377;0.05} = 1.2$, S_2^2 is the averaged value obtained by using sd^2_{ILS}, S_1^2 is for the NHIP or DR2 data. The value sd_{ILS} is the standard deviation of ILS μ_δ, sd_{NHIP} is for the NHIP μ_δ, sd_{HIP} is for the HIP μ_δ, sd_{DR2} is for the DR2 μ_δ; sd^2 is standard deviation squared in accordance with the ILS, NHIP, HIP and DR2 data. The values of F are: $F = 2.206$ if S_1^2 is for the NHIP data, and $F = 0.730$ if S_1^2 is for the DR2. In the case of the NHIP data, $F > F_{377,377;0.05}$ and the hypothesis $H_0(sd^2_{\mathrm{ILS}} = sd^2_{\mathrm{NHIP}})$ can be rejected. The same is in the case of HIP data because the NHIP astrometric data are better by up to a factor of four for stars with $V \le 8$ mag (van Leeuwen 2007). This means the values of sd^2_{ILS} are smaller than those of sd^2_{HIP} and sd^2_{NHIP}, and the ILS μ_δ values are better than the Hipparcos and new Hipparcos ones, but this is not the case for the DR2 data, even if sd^2_{ILS} is close to sd^2_{DR2}. In the case of the INDLS μ_δ values, the mean accuracy of INDLS is 0.51 mas/yr and the verification of the INDLS data using the F-test is presented in Damljanović (2020); the result is that sd^2_{INDLS} is close to sd^2_{HIP}.

The systematic errors (depending on stellar magnitude, and color index of every star) in the ILS−DR2 differences of μ_δ have been already investigated, and they are nearly at the same level of 0.1 mas/yr (Damljanović and Taris 2019). In that paper, using common 387 stars it was concluded that all compared catalogs (ILS, DR2, and NHIP) are close to each other, but with small systematic errors. This systematic part is the subject of the present paper. The reason for that systematic part was found in double or multiple stars (Vondrák et al. 1998, Vondrák 2004, Damljanović and Taris 2019), but after Lindegren's results (Lindegren 2020a,b) about a possible spin of the bright reference frame of Gaia DR2, the presented investigation is focused on two components ω_X and ω_Y which can here be obtained by using μ_δ values.

3. INVESTIGATION OF ω_X AND ω_Y OF THE BRIGHT DR2 SPIN

The 3σ criterion was applied to 387 differences ILS−DR2 two times, and two sets of rejected stars are presented in Table 2: the Hipparcos number (HIP), ILS−DR2, and ILS−HIP differences Δ of μ_δ. Each double or multiple star is marked with D. Nine stars were rejected after the first application of this criterion (the first column of Table 2), and seven stars

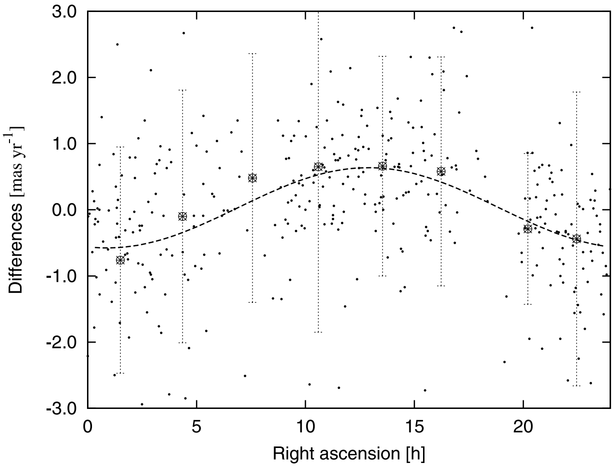

Fig. 1: Mean proper motion differences as a function of α (ILS−DR2, one rectangle per 3^{h} subinterval via α), μ_δ differences ILS−DR2 for 387 ILS stars (points), the obtained spin (together $\omega_X = 0.14$ mas/yr and $\omega_Y = -0.59$ mas/yr, dashed curve) using 378 stars and LSM.

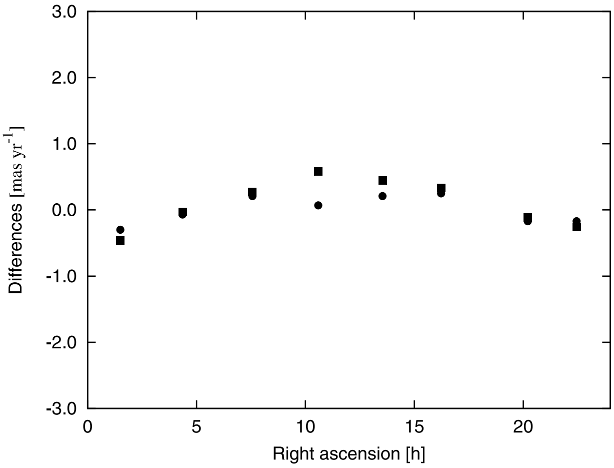

Fig. 2: Mean proper motion differences as a function of α: ILS−NHIP (black circles), and NHIP−DR2 (black rectangles).

after the second one (the second column of that table). It is interesting that the rejected stars (in Table 2) show a better accordance between the ILS and HIP catalogs than the ILS and DR2 ones. For these stars, there are big differences of μ_δ for the ILS−DR2 and HIP−DR2, much bigger than formal errors of the catalogs. Only a few stars are marked as double or multiple stars, and an explanation for this is that some Gaia DR2 proper motions data are not reliable, even with small formal errors. There is a similar situation for the INDLS rejected stars (Damljanović 2020). After application of the 3σ criterion, there are 371 ILS stars and 672 INDLS stars for investigation of spin components ω_X and ω_Y.

Using (Lindegren 2020a):

$$\Delta \approx -\omega_X \sin\alpha + \omega_Y \cos\alpha, \qquad (1)$$

we apply Eq. (2) to calculate the unknowns a, ω_X, and ω_Y using LSM. The values Δ are differences of μ_δ (ILS−DR2, INDLS−DR2, etc.), $\Delta = \mu_\delta - \mu_\delta^{'}$, where $\mu_\delta = d\delta/dt$ is the component of the proper motion in the frame C (or catalog C) and $\mu_\delta^{'} = d\delta^{'}/dt$ in the frame $C^{'}$ (Lindegren 2020a) or catalog $C^{'}$. In this way, it is possible to obtain two components of the spin between the two astrometric catalogs.

This means, the systematic errors of pairwise differences using common stars from the catalogs are calculated applying the formula:

$$\Delta = a - \omega_X \sin\alpha + \omega_Y \cos\alpha, \qquad (2)$$

where the unknowns a, ω_X, and ω_Y describe the systematic part of the μ_δ differences.

In Fig. 1, the rectangles indicate a systematic curve (the mean proper motion differences ILS−DR2 as a function of α), and these rectangles are somewhat higher in the central part of that figure (at $\alpha \approx 12^{\mathrm{h}}$). In accordance with indication that the bright reference frame of DR2 rotates relative to the faint part of DR2 (Lindegren 2020b), here two components of spin (ω_X and ω_Y) are investigated using LSM (results are in Table 1), and the obtained curve for the case of ILS−DR2 is presented in Fig. 1 as a dashed curve. The curve is following the rectangles. The rectangles are averaged differences over 3^{h} long subintervals via α (Table 3). The error bars for the plotted mean differences ILS−DR2 in Fig. 1 are given in Table 3. These error bars are suitable values of standard deviations of differences of 3^{h} long segments via α. In Fig. 1, the μ_δ differences ILS−DR2 for 371 stars are presented with points. These points are concentrated around the rectangles and obtained curve, and the curve is calculated using the values $a = 0.03$ mas/yr, $\omega_X = 0.14$ mas/yr and $\omega_Y = -0.59$ mas/yr in the case of ILS−DR2 and $n = 371$ (Table 1); the suitable residuals (in Table 3) for the case ILS−DR are close to zero which means that the curve is well determined. Other differences (ILS−NHIP and NHIP−DR2) are presented in Fig. 2, and their values with standard deviations are in Table 3. The obtained values for ILS−HIP are close to the values for ILS−NHIP because, almost always, HIP and NHIP give practically the same result. The catalogs fall in three distinct groups: DR2, HIP and NHIP, ILS and INDLS. There are some differences between the three groups, and the bright DR2 deviating most from the other two groups. The differences ILS−NHIP and INDLS−NHIP are presented just to show that their ω_Y values are smaller than the values of DR2 from the other two groups. Each set of differences ILS−DR2 and NHIP−DR2 is with the same sinusoidal shape, but just with a different value of amplitude. The related unknowns a (free term),

Table 1: Values of: s_0, free term (a), and two spin components (ω_X, ω_Y) using $n = 371$ μ_δ differences (ILS−NHIP, ILS−DR2, NHIP−DR2), 353 μ_δ differences ILS−DR2, and suitable differences for 672 INDLS stars.

Catalog, n, s_0 [mas/yr]	a [mas/yr]	ω_X [mas/yr]	ω_Y [mas/yr]
ILS−NHIP, $n = 371$, 1.02	−0.03(0.05)	0.04(0.08)	−0.18(0.07)
ILS−DR2, $n = 371$, 1.16	0.03(0.06)	0.14(0.10)	−0.59(0.08)
NHIP−DR2, $n = 371$, 1.18	0.06(0.06)	0.10(0.10)	−0.42(0.08)
ILS−DR2, $n = 353$, 0.93	0.09(0.05)	0.14(0.08)	−0.50(0.06)
INDLS−NHIP, $n = 672$, 1.48	0.05(0.06)	0.08(0.08)	−0.19(0.08)
INDLS−DR2, $n = 672$, 1.73	0.01(0.07)	−0.03(0.09)	−0.54(0.10)
NHIP−DR2, $n = 672$, 1.49	−0.03(0.06)	−0.11(0.08)	−0.35(0.08)

Table 2: Rejected ILS stars by applying the 3σ criterion, their Hipparcos number (HIP), ILS−DR2, and ILS−HIP differences Δ of μ_δ (double or multiple star is marked with D); 9 stars were rejected after the first running of that criterion (the first column), and 7 stars after the second one (the second column).

HIP (ILS−DR2, ILS−HIP) Δ [mas/yr]	HIP (ILS−DR2, ILS−HIP) Δ [mas/yr]
5465 (−7.00, −2.02)	5045 (−4.46, −2.20)
9493 (−7.25, −0.57)	17460 (−4.36, 1.56)
22279 (7.30, −1.53) D	40305 (4.27, −2.54)
44064 (6.46, 4.63) D	56613 (5.47, −0.57) D
45910 (−7.08, −0.72)	62825 (4.29, 0.15)
55060 (16.32, −0.51)	67529 (−5.98, −3.64)
62145 (8.56, 8.56)	109096 (−4.44, 1.50)
75256 (−6.21, 1.77)	
117622 (17.02, −1.00)	

ω_X, and ω_Y of Eq. (2) are calculated and presented in Table 1 (top). The results from the INDLS data are close to the results from the ILS data and are presented in Table 1 (below).

For each presented combination of catalogs (in Table 1), using 371 stars (ILS data) and 672 stars (INDLS data), the values of ω_Y are: negative as Lindegren's (2020b) and Brandt's (2018) results, significant at the 2σ level, of the order of 0.1 mas/yr as Brandt's result (2018), remarkable for ILS−DR2 and INDLS−DR2 (about −0.5 mas/yr), and relative (between two catalogs). The values of ω_X (Table 1) are not significant at the 2σ level. The value ω_Z is impossible to obtain from μ_δ data, but it is nearly zero from other investigations (Lindegren 2020a). Some values of the coefficients a, ω_X, and ω_Y (in Table 1) are small for any combination of catalogs or their standard errors are higher than the corresponding values.

Also, the sum s_0 of the random errors for both catalogs (or a combination of formal errors for pair of catalogs) is in Table 1; s_0 is the unit weight error of the solution of the system (or the sample standard deviation). It is the random part of Δ, and it is calculated as:

$$s_0 = \sqrt{\frac{1}{n-3}\sum_{i=1}^{n}(\Delta_i - A_i)^2}, \tag{3}$$

where $n = 371$ in the case of ILS data, $\Delta_i = \mu_{\delta i} - \mu'_{\delta i}$, and $A_i = a - \omega_X \sin\alpha_i + \omega_Y \cos\alpha_i$.

In the ILS−NHIP case, $s_0 \approx 1$ mas/yr, but in the ILS−DR2 ($n = 371$) and NHIP−DR2 cases, $s_0 \approx$ 1.2 mas/yr. As mentioned, the Gaia DR2 stars with $G \leq 6$ mag have inferior astrometry (Lindegren et al. 2018), and this could be the reason why $s_0 \approx 1.2$ mas/yr. It means the μ_δ values of the DR2 bright stars could be underestimated. The results from the INDLS data (Table 1) support this conclusion.

4. RESULTS AND DISCUSSION

4.1. Investigation of systematic variability using the Abbe criterion

The Abbe criterion (Malkin 2013) could be used to check the trends and low-frequency variations in Δ_i. It is aimed at testing the hypothesis that all mathematical expectancies of the analyzed Δ_i are equal. The ratio $R = a_1/a_2$ is the Abbe statistic, where a_1 is the Allan variance and a_2 is the dispersion of the data Δ_i. The value a_2 is greater than a_1 if there are trends and low-frequency variations in Δ_i values (or if $R < R_0$, where R_0 is the critical point of the Abbe distribution). If $R < R_0$, the hypothesis that there is no trend in Δ_i is rejected (or the conclusion is that there are statistically significant systematic variations in Δ_i values). The Abbe criterion is here applied to the Δ_i values, to explain variability in ILS−DR2 and other suitable differences in Table 1, i.e. whether or not some variability could be explained by formal errors. After applying that criterion to 378 stars and ILS−DR2 differences, the calculated values (for probability level of 0.05) are: $R = 0.876$, $R_0 = 0.916$, $a_1 = 1.725$, $a_2 = 1.969$. The average value $\overline{\Delta}$ for differences Δ_i is -0.011 ± 1.403

mas/yr. The values a_1 and a_2 are calculated using formulas:

$$a_1 = \frac{1}{2n-1}\sum_{i=1}^{n-1}(\Delta_{i+1} - \Delta_i)^2, \tag{4a}$$

$$a_2 = \frac{1}{n-1}\sum_{i=1}^{n}(\Delta_i - \overline{\Delta})^2. \tag{4b}$$

Because $n = 378$, R_0 is calculated using the formula $R_0 = 1 + U_q/[n + 0.5(1 + U_q^2)]^{0.5}$, where U_q is the quantile of the order q of standard (normal) distribution of values Δ_i, and for $q = 0.05$ it is $U_{0.05} = -U_{0.95} = -1.64485$. It is $R < R_0$, and we conclude that the set of values Δ_i is not possible to explain with only formal errors (in line with the Abbe criterion). It means, there is some systematic part (it could be in accordance with the spin of the bright DR2). After removing the suitable values of the obtained curve presented in Fig. 1 from ILS−DR2 differences to get residuals, and applying the Abbe criterion to these 378 residuals, there are other values: $R = 1.016$, $a_1 = 1.726$, $a_2 = 1.699$, $\overline{\Delta} = -0.014$ mas/yr. Now, it is $R \geq R_0$ and we conclude that the set of values Δ_i contains only formal errors (in line with the Abbe criterion). This means that the detected systematic part (values a, ω_X, and ω_Y of ILS−DR2 in Table 1) is well determined and removed from the ILS−DR2 differences. In accordance with the Abbe criterion at points and rectangles (differences ILS−DR2, in Fig. 1) there is some systematic part (not only formal errors). In suitable residuals that systematic part is not detected and there are only formal errors.

4.2. Investigation of normal distribution of Δ_i differences by the Shapiro-Wilk test and rejected stars by the method of Tukey's fences

The Abbe criterion is valid if the ILS−DR2 differences Δ_i satisfy the standard (normal) distribution. Fig. 3 shows the distributions of 378 ILS−DR2 differences (solid line) and of residuals (dashed line). It looks like a normal distributions but it is necessary to check it by using some additional test. The Shapiro-Wilk test (Shapiro and Wilk 1965) is applied to 378 differences Δ_i and the mentioned residuals. The null hypothesis that the population is normally distributed is rejected. On the other hand, if some of 378 values Δ_i could be rejected, it is possible to get the normal distribution. Because of this, more differences Δ_i were rejected using the method of Tukey's fences (Tukey 1977).

The method of Tukey's fences is applied to 378 differences, and 25 stars are rejected (two cases are double stars, see Table 4). After that, there are 353 stars which follow the normal distribution of Δ_i for a new calculation of a, ω_X and ω_Y (the results for ILS−DR2 and $n = 353$ are in Table 1). Now, after the Shapiro-Wilk test, both sets of data (ILS−DR2 differences and suitable residuals for $n = 353$) satisfy the normal (standard) distribution. Such a distribution is necessary for a correct application of the Abbe criterion.

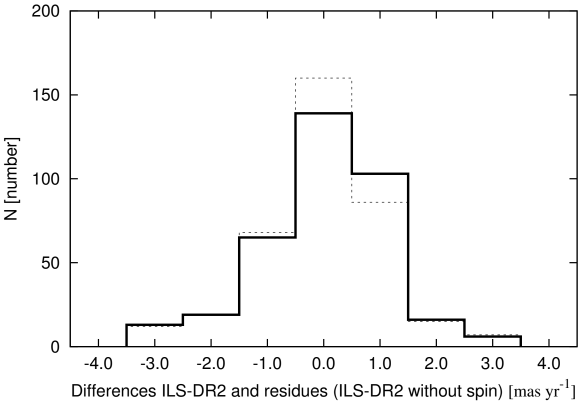

Fig. 3: Distribution of differences ILS−DR2 (solid line), and residuals (ILS−DR2 without obtained curve, dashed line) of 378 ILS stars.

4.3. Results

In the case ILS−DR2 and $n = 353$ (Table 1), the new value ω_X is stable, and for the value ω_Y there is a small change (about 15%). Now, it is -0.50 ± 0.06 mas/yr instead of -0.59 ± 0.08 mas/yr, but the standard deviation (0.06) is better than the first one (0.08 for $n = 371$ stars, Table 1). As for the Abbe criterion and method of Tukey's fences, there is a similar situation for other differences: INDLS−DR2, etc.

In Table 1, the value s_0 for ILS−DR2 is going from 1.16 mas/yr ($n = 371$) to 0.93 mas/yr ($n = 353$). As to the solution with $n = 371$ stars, this means that the 3σ criterion is applied twice: to 387 stars (9 stars are rejected) and, after that, to 378 stars (7 stars are rejected); all rejected stars are in Table 2. About the solution with $n = 353$ stars, after the first applying the 3σ criterion, there are 378 stars and the method of Tukey's fences is applied to these 378 stars to get 353 stars; 25 rejected stars are in Table 4.

Using Eq. (4), the Abbe criterion is again applied to $n = 353$ values of Δ_i (which are following the normal distribution), we can conclude that: $R < R_0$ for ILS−DR2 differences ($R_0 = 0.913$, $R = 0.854$), and $R \geq R_0$ for suitable residuals ($R_0 = 0.913$, $R = 1.008$). We can not explain the 353 ILS−DR2 differences Δ_i using only formal errors, but we can do it it for the suitable residuals. It means, the curve (in Fig. 1) is well detected in the mentioned differences and removed from them to get suitable residuals. Even with $n = 353$ stars, there are no remarkable changes of the results presented in Table 1. It is

Table 3: Average values $\overline{\Delta}$ with standard deviations sd of μ_δ differences Δ (ILS−NHIP, ILS−DR2, NHIP−DR2) and residuals (ILS−DR2 without obtained spin), m is the number of ILS stars in 3^{h} long subinterval via α.

Subinterval [h]	m	$\overline{\Delta}(sd)$ [mas/yr]	$\overline{\Delta}(sd)$ [mas/yr]	$\overline{\Delta}(sd)$ [mas/yr]	$\overline{\Delta}(sd)$ [mas/yr]
		Catalog ILS−NHIP	Catalog ILS−DR2	Residuals ILS−DR2	Catalog NHIP−DR2
$[0,3)$	67	-0.30(0.90)	-0.76(1.71)	-0.20(1.71)	-0.46(1.85)
$[3,6)$	42	-0.07(1.24)	-0.10(1.91)	0.24(1.90)	-0.03(2.16)
$[6,9)$	22	0.21(1.64)	0.48(1.88)	0.34(1.85)	0.27(1.74)
$[9,12)$	64	0.07(0.93)	0.65(2.50)	0.14(2.48)	0.58(2.54)
$[12,15)$	56	0.21(1.35)	0.66(1.66)	0.05(1.66)	0.45(1.17)
$[15,18)$	41	0.25(1.43)	0.58(1.73)	0.17(1.76)	0.33(2.19)
$[18,21)$	29	-0.17(1.09)	-0.29(1.14)	-0.11(1.16)	-0.11(0.68)
$[21,24)$	66	-0.17(0.96)	-0.44(2.22)	0.01(2.23)	-0.26(2.20)

possible to conclude that the curve (in Table 1, and in Fig. 1) is well calculated, and the procedure for obtaining two spin components ω_X and ω_Y is correct. Also, in all catalogs (in Table 1) there is the rotation ω_Y which is of the order of 0.1 mas/yr for stars between 4 and 8 mag in the V domain, but $\omega_Y \approx -0.5$ mas/yr is remarkable in the ILS−DR2 and INDLS−DR2 cases. The results for INDLS data (Table 1, bottom) support those for ILS data (Table 1, top).

A few possible sources for the observed differences are marked: a distortion in the Gaia DR2 positional reference frame, a distortion in the Gaia DR2 proper motions, or a distortion in the Hipparcos positional reference frame; or any combination of these is possible. The ILS and INDLS stars were reduced to the Hipparcos reference system. In this paper, a distortion in the Gaia DR2 proper motions (via the spin of the bright DR2) is marked as a serious candidate for the detected systematic error.

Table 4: Rejected 25 ILS stars by applying the method of Tukey's fences, their Hipparcos numbers (HIP), and ILS−DR2 differences Δ of μ_δ; every double or multiple star is marked with D.

HIP (ILS−DR2) Δ [mas/yr]	HIP (ILS−DR2) Δ [mas/yr]
4584 (−3.32)	56613 (5.47) D
5045 (−4.46)	62825 (4.29)
5544 (−3.94)	67529 (−5.98)
7370 (−4.01)	75543 (3.61)
11611 (−2.94)	75825 (−2.73)
17460 (−4.36)	79236 (−3.30)
17475 (−2.79)	84606 (3.53) D
19030 (−3.13)	100651 (−3.08)
20241 (3.51)	106306 (−4.03)
20933 (−2.85)	109096 (−4.44)
40305 (4.27)	113505 (−3.97)
54063 (4.02)	113766 (−3.97)
	115317 (3.62)

5. CONCLUSIONS

Following an indication (Lindegren 2020b) that the bright reference frame of Gaia DR2 rotates relative to the faint part of DR2, the independent ILS and INDLS data of μ_δ are used to investigate two spin components (ω_X and ω_Y). We can not calculate the component ω_Z using only μ_δ data, but $\omega_Z \approx 0$ mas/yr from other investigation (Lindegren 2020a). The original ILS catalog of μ_δ is based on the ground-based observations during the period from 1899.7 to 1979.0; it is about 80 years (or about 90 years with the Hipparcos point for the moment 1991.25). There was a network of seven ILS instruments (zenith telescopes at latitude $39\overset{\circ}{.}1$). These data provide accurate ILS μ_δ values of 387 common ILS and Hipparcos stars. The original method was applied (Damljanović and Taris 2019), and the mean accuracy of the ILS μ_δ is about 0.21 mas/yr (0.51 mas/yr for INDLS data) which is in accordance with modern astrometry. The original INDLS catalog is based on observations of seven independent latitude stations over many decades during the last century (Damljanović 2020). Both catalogs (ILS and INDLS) are referred to the Hipparcos reference frame, but their merit is the long time baseline ($\Delta t \approx 90$ years) important for μ_δ. The short time baseline ($\Delta t < 4$ years of the Hipparcos data) could be the reason why the HIP and NHIP μ_δ values are affected by the systematic errors of double and multiple stars.

After the 3σ criterion, we continue with 371 ILS stars (and with 672 INDLS stars) to investigate the ω_X and ω_Y spin components of the bright Gaia DR2 reference frame (stars with $G \leq 13$ mag). Two mentioned components are obtained and presented in Table 1. The value ω_X is not significant at the 2σ level but $\omega_Y = -0.59 \pm 0.08$ mas/yr for ILS$-$DR2 (-0.54 ± 0.10 mas/yr for INDLS$-$DR2). It is about -0.20 ± 0.07 for ILS$-$NHIP, about -0.40 ± 0.08 for NHIP$-$DR2; some amount of the order of 0.1 belongs to each catalog but DR2 is deviating most from the other two catalog groups (HIP and NHIP, ILS and INDLS). The results for ILS and INDLS data are close to each other (Table 1). The direction is the same as Lindegren's (2020b) result, but the presented result is larger than Lindegren's one. Other values (of a and ω_X in Table 1) are small for any combination of catalogs or their standard errors are higher than the corresponding values. Also, some stars are rejected using the method of Tukey's fences to get the normal distribution of Δ_i differences, the normal distribution of differences is checked with the Shapiro-Wilk test, and the Abbe criterion (useful for the normal distribution of differences) is applied to Δ_i and suitable residuals. All presented calculations and investigations show that the obtained curve (in Fig. 1) and results (Table 1) are well determined and in accordance with ω_Y. The ILS results (Table 1, top) are close to the results from the INDLS data (Table 1, bottom).

The values of ω_Y (Table 1, top) are: negative as other results (Lindegren 2020b, Brandt 2018), significant at the 2σ level, the rotation is of the order of 0.1 mas/yr as Brandt's result (2018) for DR2, that rotation is a relative one (between two catalogs), and it is obtained using the bright stars with the V magnitude from 4 to 8. The same conclusions are obtained using the INDLS data (Table 1, bottom).

The value $s_0 = 0.93$ mas/yr for ILS$-$DR2 ($n = 353$) is smaller than 1.16 ($n = 371$) because of rejected stars using the method of Tukey's fences. The inferior astrometry of DR2 stars with $G \leq 6$ mag (Lindegren et al. 2018) could be the reason for smaller value of s_0 in the case ILS$-$NHIP ($s_0 \approx 1.0$ mas/yr) than ILS$-$DR2 and NHIP$-$DR2 ($s_0 \approx 1.2$), and it could be concluded that the μ_δ values of the bright DR2 are underestimated. Also, that the conclusion is in accordance with the INDLS data.

The presented results are in line with the activity of the IAU Working Group on Astrometry by Small Ground-Based Telescopes.

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[4] `ftp://cdsarc.u-strasbg.fr/pub/cats/J/AN/341/8`

РОТАЦИЈА GAIA DR2 РЕФЕРЕНТНОГ СИСТЕМА СЈАЈНИХ ЗВЕЗДА КОРИСТЕЋИ НЕЗАВИСНУ АНАЛИЗУ μ_δ ПОДАТАКА ИЗ ILS И INDLS КАТАЛОГА

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УДК 521.96 ILS, INDLS, Gaia DR2
Оригинални научни рад

Gaia DR2 референтни систем би требало да буде без релативне ротације у односу на квазаре и сагласан са International Celestial Reference System (ICRS). То је постигнуто за слабе DR2 звезде (за које важи да је Gaia магнитуда $G \geq 16$) преко директних Gaia посматрања квазара ($G \geq 17$ mag), али за сјајне DR2 звезде ($G \leq 13$ mag) није и због тога се јавља релативна ротација сјајног Gaia DR2 дела у односу на слаби DR2 део. Додатно, астрометрија врло сјајних DR2 звезда ($G \leq 6$ mag) је лошија у односу на остале звезде. Због тога, израчунали смо две компоненте поменуте ротације (ω_X, ω_Y) користећи два независна каталога (ILS са 387 звезда и INDLS са 682 звезда) базирана на вишедеценијским ширинским посматрањима звезда. Оба су у Hipparcos референтном систему и V магнитуда им је од 4 до 8 (критичан део за DR2). Користили смо и Hipparcos податке нове обраде (NHIP), мада је предност ILS и INDLS у вишедеценијском материјалу ($\Delta t \approx 90$ година) а то је важно за μ_δ јер је стандардна грешка μ_δ пропорционална $1/\Delta t$. Примењена је метода најмањих квадрата на разлике два каталога (ILS$-$DR2, INDLS$-$DR2, итд) и добили смо вредности које потврђују поменуту ротацију. У раду су коришћене одговарајуће статистике (метод Tukey's fences, Abbe-критеријум, Shapiro-Wilk тест, итд). Добијена вредност за ω_Y је значајна на 2σ нивоу, и ILS и INDLS каталози се могу користити за проверу сјајног дела Gaia DR2.